Let γ_n be the rectangular path

$$[n + 1/2 + ni, -n - 1/2 + ni, -n - 1/2 - ni, n + 1/2 - ni, n + 1/2 + ni].$$

Evaluate the integral

$$\pi\int_{\gamma_n} (z+a)^2 \cot(\pi z)dz$$

for an integer $a \neq 0$. Conclude that

$$\lim_{n\to\infty} \pi\int_{\gamma_n} (z+a)^2 \cot(\pi z)dz = 0.$$

and that

$$\frac{\pi^2}{\sin^2 \pi a} = \sum_{n=-\infty}^{\infty} \frac{1}{(a+n)^2}$$

(Hint: Use the fact that $|\cot \pi z| \leq 2$ for $z \in \gamma_n$ and n sufficiently large). Conclude that

$$\frac{\pi^2}{8} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$