1. Show that $X$ and $Y$ are connected topological spaces if and only if $X \times Y$ is connected.

2. If $X$ is a topological space and $Y \subseteq X$, then we can define a topology on $Y$ as follows: Open subsets of $Y$ are the intersection of open subsets of $X$ with $Y$. Check that this really defines a topology on $Y$.

3. Show that open subsets of $\mathbb{Q}$ are unions of open intervals $(a, b)$ for $a, b \in \mathbb{Q}$.

4. Let $X$ be a topological space. Let $Y \subseteq X$. Show that if $Y$ is connected then so is $\overline{Y}$.

5. Let $X$ be a topological space. $(A_i)_{i \in I}$ is connected space of $X$. Suppose that for all $i, j$ $A_i \cap A_j \neq \emptyset$. Show that $\bigcup A_i$ is connected.