

**20.03.2006-24.03.2006**

**HW 152**

1. Show that  $X$  and  $Y$  are connected topological spaces if and only if  $X \times Y$  is connected.
2. If  $X$  is a topological space and  $Y \subseteq X$ , then we can define a topology on  $Y$  as follows: Open subsets of  $Y$  are the intersection of open subsets of  $X$  with  $Y$ . Check that this really defines a topology on  $Y$ .
3. Show that open subsets of  $\mathbb{Q}$  are unions of open intervals  $(a, b)$  for  $a, b \in \mathbb{Q}$ .
4. Let  $X$  be a topological space. Let  $Y \subseteq X$ . Show that if  $Y$  is connected then so is  $\bar{Y}$ .
5. Let  $X$  be a topological space.  $(A_i)_{i \in I}$  is connected space of  $X$ . Suppose that for all  $i, j$   $A_i \cap A_j \neq \emptyset$ . Show that  $\cup A_i$  is connected.