# Math 151
# Resit

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Justify all your answers. A nonjustified answer will not receive any grade whatsoever, even if the answer is correct. DO NOT use symbols such as ∀, ∃, ⇒. Make full sentences with correct punctuation.

1. Show that the sum of the reciprocals of natural numbers whose decimal expansion contains at least a zero $1/10 + \ldots + 1/90 + 1/100 + 1/101 + \ldots + 1/109 + 1/110 + 1/120 + \ldots$ diverges.
2. Show that if $\sum_{i=0}^{\infty} a_i$ converges then $\lim_{i\to\infty} a_i = 0$.
3. Show that a sequence $\sum_{i=0}^{\infty} a_i$ converges if and only if for all $\epsilon > 0$, there exists an $N$ such that for all $n > m > N$, $|\sum_{i=m}^{n} a_i| < \epsilon$.
4. Suppose that $u_n, v_n > 0$ and $u_{n+1}/u_n \le v_{n+1}/v_n$ eventually. If $\sum_n v_n$ converges then $\sum_n u_n$ converges.
5. Let $a_n > 0$, $b_n > 0$. Show that if $\lim_{i\to\infty} b_i/a_i = 0$ and $\sum_{i=0}^{\infty} a_i$ converges then $\sum_{i=0}^{\infty} b_i$ converges as well.
6. Show that an absolutely convergent series is convergent.
7. Let $\sum_{i=0}^{\infty} a_i$ be an absolutely convergent series. Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be a bijection. Let $b_i = a_{f(i)}$. Show that $\sum_{i=0}^{\infty} b_i$ is also absolutely convergent and its sum is equal to $\sum_{i=0}^{\infty} a_i$.
8. Let us partition the terms $(a_i)_i$ of an absolutely convergent series $\sum_{i=0}^{\infty} a_i$ in two disjoint and infinite subsets $(b_i)_i$ and $(c_i)_i$. Show that $\sum_{i=0}^{\infty} b_i$ and $\sum_{i=0}^{\infty} c_i$ are absolutely convergent series and $\sum_{i=0}^{\infty} a_i = \sum_{i=0}^{\infty} b_i + \sum_{i=0}^{\infty} c_i$.
9. Show that the sum of the reciprocals of natural numbers whose decimal expansion does not contain a zero $1/1 + \ldots + 1/9 + 1/11 + + \ldots + 1/19 + 1/21 + \ldots$ converges.