Math 151  
Exercises (quiz)  

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Justify all your answers. A nonjustified answer will not receive any grade whatsoever, even if the answer is correct. DO NOT use symbols such as ∀, ∃, ⇒. Make full sentences with correct punctuation.

1. Let \((a_n)_n\) be a convergent sequence of real numbers.  
a. Does the sequence \((a_{2n})_n\) converge necessarily?  
b. Assume \(a_n \neq 0\) for all \(n\). Does the sequence \((a_n/a_{n+1})_n\) converge necessarily?  

2. Let \((a_n)_n\) be a convergent sequence of real numbers. Suppose that \(a_n \in \mathbb{Z}\) for all \(n\). Prove or disprove: \(\lim_{n \to \infty} a_n \in \mathbb{Z}\).  

3. Let \((a_n)_n\) be a convergent sequence of real numbers. Suppose that \(5a_n/2 \in \mathbb{N}\) for all \(n\). What can you say about \(\lim_{n \to \infty} a_n\)?  

4. Let \((a_n)_n\) be a sequence of real numbers such that the sequence \((a_{2n})_n\) converges to 0. Does the sequence \((a_n)_n\) converge necessarily?  

5. Find the following limits and prove your result using only the definition.  
a. \(\lim_{n \to \infty} \frac{2n-5}{5n+2}\)  
b. \(\lim_{n \to \infty} \frac{2n^2-5}{6n+2}\)  
c. \(\lim_{n \to \infty} \frac{2n^2-5}{n+2}\)  

**Note:** A sequence \((a_n)_n\) of real numbers is said to converge to infinity if for all \(A\) there is an \(N\) such that if \(n > N\) then \(a_n > A\).

6. Let \((a_n)_n\) be a sequence of real numbers such that \(\lim_{n \to \infty} a_n = \infty\). Show that \(\lim_{n \to \infty} a_{2n} = \infty\)?  

7. Let \((a_n)_n\) be a sequence of nonnegative real numbers. Suppose that the sequence \((a_n^2)_n\) converges to \(a\). Show that the sequence \((a_n)_n\) converges to \(\sqrt{a}\).