Math 151
Exercises (quiz)

Fall 2005
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December 21, 2005

Justify all your answers. A nonjustified answer will not receive any grade whatsoever, even if the answer is correct. DO NOT use symbols such as $\forall$, $\exists$, $\Rightarrow$. Make full sentences with correct punctuation.

1. Let $(a_n)_n$ be a convergent sequence of real numbers.

   a. Does the sequence $(a_{2n})_n$ converge necessarily?

   b. Assume $a_n \neq 0$ for all n. Does the sequence $(a_n/a_{n+1})_n$ converge necessarily?

2. Let $(a_n)_n$ be a convergent sequence of real numbers. Suppose that $a_n \in \mathbb{Z}$ for all n. Prove or disprove: $\lim_{n \to \infty} a_n \in \mathbb{Z}$.

3. Let $(a_n)_n$ be a convergent sequence of real numbers. Suppose that $5a_n/2 \in \mathbb{N}$ for all n. What can you say about $\lim_{n \to \infty} a_n$?

4. Let $(a_n)_n$ be a sequence of real numbers such that the sequence $(a_n^2)_n$ converges to 0. Does the sequence $(a_n)_n$ converge necessarily?

5. Find the following limits and prove your result using only the definition.

   a. $lim_{n \to \infty} \frac{2n-5}{5n+2}$

   b. $lim_{n \to \infty} \frac{2n^2-5}{-5n+2}$

   c. $lim_{n \to \infty} \frac{2n^2-5}{n^3+2}$

   Note: A sequence $(a_n)_n$ of real numbers is said to *converge to infinity* if for all A there is an N such that if $n > N$ then $a_n > A$.

6. Let $(a_n)_n$ be a sequence of real numbers such that $\lim_{n \to \infty} a_n = \infty$. Show that $\lim_{n \to \infty} a_{2n} = \infty$?

7. Let $(a_n)_n$ be a sequence of nonnegative real numbers. Suppose that the sequence $(a_n^2)_n$ converges to a. Show that the sequence $(a_n)_n$ converges to $\sqrt{a}$.