

**Summer School
Calculus Homework
July 2006
Ali Nesin**

Curve Sketching

1. Study the curve defined by $(x^2 + y^2)^2 - x^3 - 3xy^2 = 0$.
2. Study the curves defined by $(x - y)(x^2 + y^2)^3 = 3a^2\ell^2x^3$ for different values of $a > 0$ and $\ell > 0$. (Find the asymptote(s)).
3. Study the curves defined by the polar equation $r = 2a \frac{\sin n\theta}{\sin(n-1)\theta}$ for different values of $a > 0$ and $n \in \mathbb{N}^{>1}$. (Find the asymptote(s)).
4. Study the curves whose polar equations are $r^3 = \ell^3 \cos 3\theta$ for different values of $\ell > 0$.
5. Study the curves defined by $r = \frac{a\ell \sin \theta}{a - \ell \cos \theta}$ for different values of $a > 0$ and $\ell > 0$. (Find the asymptote(s)).
6. Study the curves defined by $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$ for different values of $a > 0$ and $b > 0$.

Integration

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| 1. $\int \frac{x^2 - x + 2}{x^5 + 2x^3 + 3x^2} dx$ | 9. $\int \frac{\sin 2x}{\cos 3x} dx.$ | 17. $\int \frac{\cos^p x}{\sin^q x} dx.$ |
| 2. $\int \frac{dx}{x^4 - x^3 + x - 1}.$ | 10. $\int \frac{\sin x dx}{1 + \cos x + \cos 2x}.$ | 18. $\int \frac{\cos^3 x - \sin^2 x}{2 + \cos x} dx$ |
| 3. $\int \frac{x^2 dx}{1 - x^4}.$ | 11. $\int \frac{dx}{\cos^2 x + a^2 \sin^2 x}.$ | 19. $\int \frac{\cos^2 x + 3 \cos x - 1}{\cos^3 x + 5 \cos^2 x + 8 \cos x + 4} dx$ |
| 4. $\int \frac{x^5 dx}{(x^2 + 1)^3}.$ | 12. $\int \frac{1 - \cos(x/3)}{\sin(x/2)} dx.$ | 20. $\int x^m (a + bx^n)^p dx.$ |
| 5. $\int \frac{ax^3 + bx^2 + cx + d}{(x^2 + 1)^2} dx.$ | 13. $\int \frac{dx}{\sin^4 x + \cos^4 x}.$ | 21. |
| 6. $\int \frac{x^9 dx}{(1 + x^5)^3}.$ | 14. $\int \frac{dx}{\sin^3 x + \cos^3 x}.$ | |
| 7. $\int \frac{1 + 2x - 3x^2}{x - x^3} dx.$ | 15. $\int \frac{\cos x dx}{\sin x + \cos x}.$ | |
| 8. $\int \frac{\cos^3 x}{1 - 2 \sin x} dx.$ | 16. $\int \frac{dx}{\sin^q x \cos^p x}.$ | |

Differential Equations

$$1. yy' = \frac{5}{(x-2)(x^2 - 2x + 5)}.$$

$$2. y' \left(\frac{2y^3 - 3y^2 - y + 5}{y^2 - y - 2} \right) = \frac{x^3 - x + 1}{2x^4 - 7x^3 + 9x^2 - 5x + 1}.$$