**Math 152 Final Exam**
**2005**
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**1.** Integrate:

$\int_0^1 \frac{dx}{x^{1/3}(1+x)}$, $\int \frac{xdx}{2x^2+5x+2}$, $\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}dx$, $\int 2^x\sqrt{1+4^x}dx$, $\int \frac{x^2dx}{(3+5x^2)^{3/2}}$,

$\int \frac{x\ln(1+x^2)}{1+x^2}dx$, $\int \sin^2 x\cos^4 xdx$, $\int \frac{e^{1/x}}{x^2}dx$, $\int e^x(1-e^{2x})^{5/2}dx$,

$\int \frac{dx}{\tan x+\sin x}$, $\int \sqrt{1+e^x}dx$, $\int xe^x\cos xdx$, $\int \frac{x^4dx}{x^3-8}$, $\int \frac{x^3+x-2}{x^2-7}dx$,

$\int \frac{xdx}{\sqrt{3-4x-4x^2}}$, $\int \frac{x^2+1}{x^4+x^2+1}dx$, $\int \frac{x^2+1}{x^4+1}dx$,

**2.** Evaluate the integrals or show they diverge:

$\int_1^\infty \frac{1}{x+x^3}dx$, $\int_0^1 \sqrt{x}\ln xdx$, $\int_{-1}^1 \frac{dx}{x\sqrt{1-x^2}}$.

**3a.** Find a reduction formula for $\int (1-x^2)^n dx$.

**3b.** Show that if $n$ is a positif integer, then

$$\int_0^1 (1-x^2)^n dx = \frac{2^{2n}(n!)^2}{(2n+1)!}.$$

**3c.** Evaluate $\int (1-x^2)^{-3/2}dx$.

**4.** Let $I_{m,n} = \int_0^1 x^m(\ln x)^n dx$.

**4a.** Show that $I_{m,n} = (-1)^n\int_0^\infty x^n e^{-(m+1)x}dx$.

**4b.** Show that $I_{m,n} = \frac{(-1)^n n!}{(m+1)^{n+1}}$.

**5.** Let $I_n = \int_0^1 x^n e^{-x}dx$.

**5a.** Show that $0 < I_n < \frac{1}{n+1}$,. and hence that $\lim_{n\to\infty} I_n = 0$.

**5b.** Show that $I_n = nI_{n-1} - 1/e$.

**5c.** Show that $I_n = n!\left(1-\frac{1}{e}\sum_{j=0}^n \frac{1}{j!}\right)$.

**5d.** Deduce from a and c that $\sum_{j=0}^\infty 1/j! = e$.

**6.** The region $R$ bounded by $y = e^{-x}$ and $y = 0$ and lying to the right of $x = 0$ is rotated (a) around the $x$-axis and (b) around the $y$-axis. Find the volume of the solid of revolution generated in each case.

**7.** **a.** Find the area of an ellipse whose equation is $x^2/a^2 + y^2/c^2 = 1$.
**b**. Find the volume generated by the above ellipse when it is rotated around the $x$ axis.

**c.** "Attempt" to find the circumference of the above ellipse and show the difficulty.

**8.** A disk of radius $a$ has center at the point $(b, 0)$ where $b > a > 0$. The disk is rotated about the $y$-axis to generate a **torus**. Find its volume.

**9.** Sketch the curve given by $x = t^3 - 3t - 2$, $y = t^2 - t - 2$.

**10.** Sketch the curve given by $x = a \cos^3 t$ , $y = a \sin^3 t$ where $a > 0$ is a constant.

**11.** Find the length of the curve given by $x = e^t \cos t$, $y = e^t \sin t$ $(0 \le t \le 2)$.

**12.** Sketch the graph of the function $f(x) = \dfrac{x^4 + x^2}{x^4 + 1}$.