Analysis II (Math 152)
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1. Find $\lim_{n\to\infty} \frac{3-2n+n^2}{4+3n-2n^2}$ and $\lim_{x\to 3} \frac{x^2-2x-3}{x^2-x-6}$ and prove your results by using the definitions.

2. Suppose $\lim_{n\to\infty} x_n = x = \lim_{n\to\infty} y_n$. Let $z_{2n} = x_n$ and $z_{2n+1} = y_n$. Show that $\lim_{n\to\infty} z_n = x$ by using the definition of limits.

3. Suppose $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y \neq 0$. Is it true that $\lim_{n\to\infty} x_n/y_n = x/y$? Prove or disprove.

4. Supose $(a_n)_n$ is a sequence converging to 0. Show that if $(b_n)_n$ is a bounded sequnce. then the sequence $(a_n b_n)_n$ converges to 0. Does the converse hold?

5. Let $\alpha \in \mathbb{R}$. Is there a sequence $(a_n)_n$ of natural numbers such that $\sum_{n=1}^{\infty} \frac{a_n}{n} = \alpha$?

6. Prove or disprove for any sequence $(a_n)_n$.

6a) If $\sum_{n=0}^{\infty} a_n$ converges then $\lim_{n\to\infty} a_n = 0$.

6b) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ converges.

6c) If $\lim_{n\to\infty} a_n^2 = 0$ then $\sum_{n=0}^{\infty} a_n$ converges.

6d) If $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} a_n^2$ converges?

7. Discuss the convergence of the series.

7a) $S_r(x) = \sum_{n=0}^{\infty} n^r x^n$ for a real number r and for $x \geq 0$.

7b) $S_k(x) = \sum_{n=0}^{\infty} \binom{n+k}{n} x^n$ for a natural number k and for $x \neq 1$.

7c) $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2+1}}$ for $x \geq 0$.

7d) $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^3+1}}$.

8. Suppose $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ exists. Show that f is continuous.

9. Discuss the convergence of $\frac{xy}{x^2+y^2}$ when x and y both go to zero/infinity.