

## Math 122

2003 Spring

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1) Find the limits of the given functions and prove your answers using the  $\epsilon$ - $\delta$  definition of limit. (10 points)

a)  $\lim_{x \rightarrow -3} \frac{x+3}{x-1}$

b)  $\lim_{x \rightarrow 1} \frac{2x^4 - 6x^3 + x^2 + 3}{x-1}$

2) Find the limits of the given functions (justify your answers, 6 points).

a)  $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$

b)  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$

c)  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

3) Differentiate (4 points)

$$f(x) = \begin{cases} \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

4) Let  $(x_n)_n$  be a convergent sequence in a metric space  $(X, d)$  with  $a = \lim_{n \rightarrow \infty} x_n$ . Show that the set  $\{x_n : n \in \mathbb{N}\} \cup \{a\}$  is compact. (12 points)

5) Sketch the graphs of the following functions:

a)  $f(x) = \frac{2 - \cos x}{2 \cos x - 1}$

b)  $g(x) = \arctan \frac{x}{1-x}$

(20 points)

6) Determine the graph of parametric curve  $x(t) = t^2$ ,  $y(t) = t^3 - 3t$  where  $-2 \leq t \leq 2$ . (10 points)

7) Prove the inequalities  $\frac{1}{n} \sum_{k=1}^n \frac{n^2}{n^2 + k^2} \leq \frac{\pi}{4} \leq \frac{1}{n} \sum_{k=0}^{n-1} \frac{n^2}{n^2 + k^2}$ . **Hint:** Integrate  $\frac{1}{1+x^2}$  from

0 to 1. (6 points)

8) Prove that  $\frac{b-a}{1+b^2} \leq \arctan b - \arctan a \leq \frac{b-a}{1+a^2}$  if  $a \leq b$ . **Hint:** Apply the Mean Value

Theorem. (6 points)

9) Find

a)  $\frac{d}{dx} \int_{x^2}^{x^3} (t^2 + t + 1) dt$  (with respect to  $x$ )

b)  $\frac{d}{dt} \int_x^{x^2} \cos t^2 dt$  (with respect to  $t$ )

(6 points)

10) Find  $\frac{dy}{dx}$  if

a)  $y = x^{x^x}$

b)  $e^y = x + y$

(6 points)

11) Evaluate the following integrals (24 points)

a)  $\int \frac{8x-3}{\sqrt{1-2x-x^2}} dx$

b)  $\int \sin^3 5x dx$

c)  $\int \frac{2x+3}{16+13x^2} dx$

d)  $\int \frac{\sin x}{\cos^2 x + 8 \cos x + 17} dx$

e)  $\int \frac{\sqrt{4x^2-9}}{x} dx$

f)  $\int x\sqrt{x-1} dx$