

## Set Theory

Math 111

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**I.** Let  $X$  be any set. Show that there is no one-to-one correspondence between  $X$  and the power set  $\mathcal{P}(X)$  of  $X$ .

**II.** Show that there is no set containing all the ordinals.

**III.1.** Let  $r \in \mathbf{R}$  be such that  $|r| < 1$ . Show that  $\sum_{i=0}^{\infty} r^i$  converges, i.e. that

$$\lim_{n \rightarrow \infty} \sum_{i=0}^n r^i \text{ exists.}$$

**III.2.** Let  $(a_n)_n$  be a sequence of strictly positive real numbers. Assume that for some  $r \in (0, 1)$ ,  $a_{n+1}/a_n \leq r$ . Show that  $\sum_{i=0}^{\infty} a_i$  converges.

**III.3.** Let  $x \in \mathbf{R}$ . Show that  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$  converges.

**III.4.** (Decimal expansion of reals). Let  $r > 0$  be a real number. Show that there are integers  $a_n$  such that  $a_n \in \{0, 1, \dots, 9\}$  for  $n \geq 1$  and  $r = \sum_{n=0}^{\infty} a_n 10^{-n}$ .