Metamathematics of Elementary Mathematics

Lecture 6

Human Dimension of Mathematics

Alexandre Borovik

Nesin Mathematics Village, Şirince

Sunday 27 July 2008

1. Mathematics and the brain

Why does the mirror changes left and right . . . but does not change up and down?

Answer: it does not.

Changing of left and right is just a popular myth.

Question: Why does the myth exist?

Answer: Because we attribute to the mirror the intrinsic bilateral symmetry of our mind.

Mirror writing

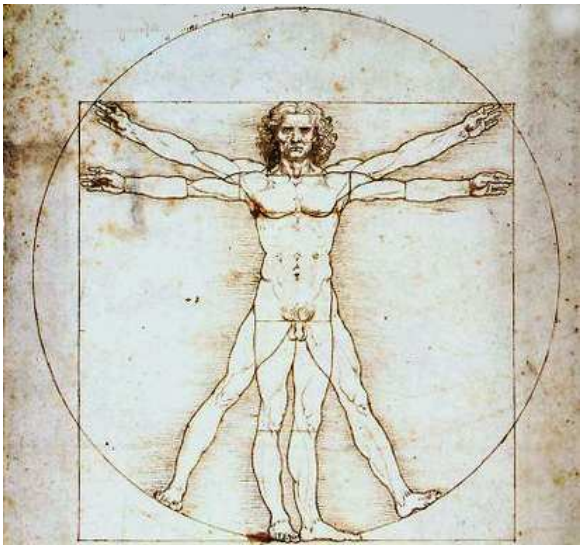
1 in 600 people can write by non-dominant hand in mirror script.

The rest 599 can do a weaker version of mirror writing:

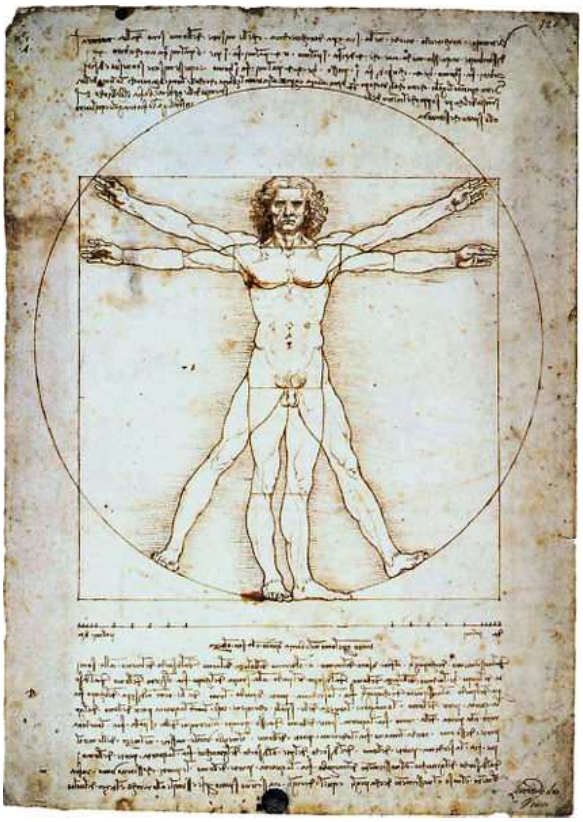
Mirror reflection:

Mirror writing: Leonardo da Vinci

Everyone know his famous study of the symmetry of human body:



It is less known that Leonardo's notes are in **mirror writing**:



Mathematical illustration: Euler's Theorem

> If an orientation-preserving isometry of the affine Euclidean space $\mathbb{AR}^3$ has a fixed point then it is a rotation around some axis.

Numerous psychological experiments show that Euler's Theorem is hardwired in our brain.

Coxeter's proof, verbatim:

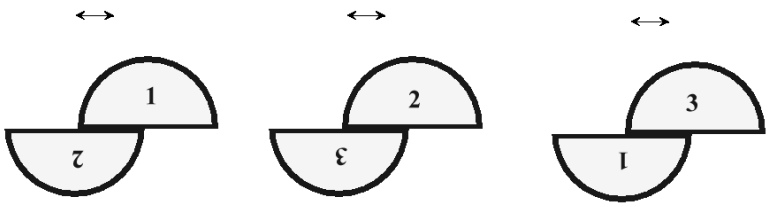
> In three dimensions, a congruent transformation that leaves a point $\mathbf{O}$ invariant is the product of at most three reflections: one to bring together the two x-axes, another for the y-axes, and a third (if necessary) for the z-axes.
>
> Since the product of three reflections is opposite, a direct transformation with an invariant point $\mathbf{O}$ can only be the product of reflections in *two* planes through $\mathbf{O}$, i.e., a rotation.

The intuition of symmetry is rooted in both visual and sensorimotor systems.

Another example of such overlap: convexity.

A symptom of interaction between sensorimotor and visual intuitions: **proof by handwaving**.

Grinding flat mirrors



Take three pieces of glass and grind 1-st and 2-nd pieces together. Then do the same for the 2-nd and the 3-rd pieces and then for the 3-rd and 1-st pieces.

Repeat many times and all three pieces of glass will become very accurately flat. **Why?**

A rather deep mathematics is just one step away:

What surface do we get if we grind only *two* pieces of glass?

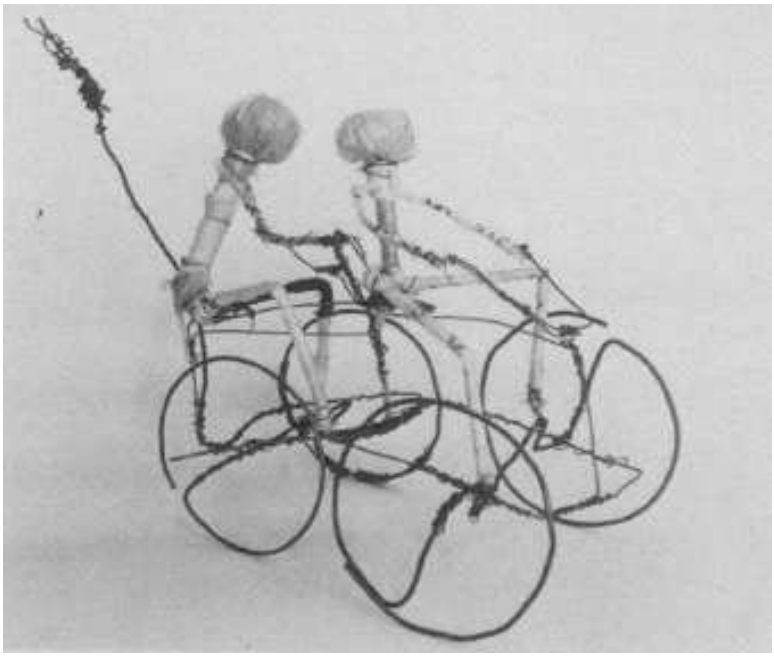
A spherical stone vase, Ancient Egypt. © Petrie Museum, UCL.

The vase was made by polishing a piece of stone. Its spherical shape is a consequence of a generalisation of Euler's theorem:

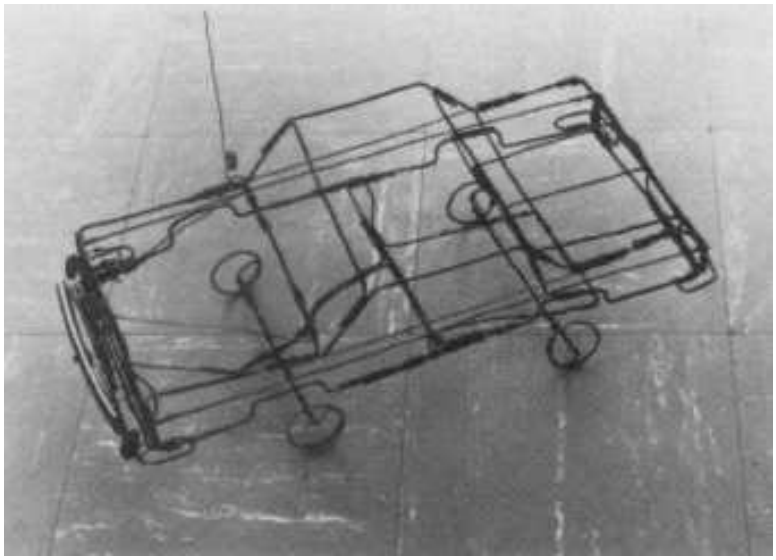
a more subtle classification of subgroups in the group of movements of the 3-dimensional space.

2. “Invisible mathematical culture”

A child in Zimbabwe, 1980s, pushing a wire toy automobile.



A wire toy pedal vehicle.



A wire toy automobile.

3. Mathematics in the Society

English banknote with a portrait of Adam Smith

The words on the note:

"The division of labour in pin manufacturing"

AN

INQUIRY

INTO THE

Nature and Cauſes

OF THE

WEALTH OF NATIONS.

By ADAM SMITH, LL. D. and F. R. S.
Formerly Profeſſor of Moral Philoſophy in the Univerſity of GLASGOW.

IN TWO VOLUMES.

VOL. I.

LONDON:

PRINTED FOR W. STRAHAN; AND T. CADELL, IN THE STRAND.

MDCCLXXVI.

Book I, Chapter I: Of The Division of Labour

On pin manufacturing:

> "One man draws out the wire; another straights it; a third cuts it; a fourth points it; a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on is a peculiar business; to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations."

Adam Smith's conclusion:

Separation of the pin production process into 18 operations increases the productivity by factor of **240**.

The history of Western civilisation is the history of ever deepening division of labour.

The history of Western civilisation is the history of ever deepening division of labour.

And we reached a unique point in history when 95% of people have no vaguest idea about the working of 95% of technology in their immediate use.

The consequences are profound.

Mathematics built-in in a mobile phone or MP3 player is beyond understanding by most graduates from mathematics departments in British universities.

In the emerging division of **intellectual labour**, mathematics is a 21st century equivalent of sharpening a pin.

Of course, the same is true about physics, chemistry, biology ...

... although biology is perhaps not sharpening the pin but attaching a head, which, as Adam Smith remarks, in itself consists of two or three operations.

We have to admit that 95% of population do not need any mathematics beyond use of a calculator.

But what are the implications for mathematical education?

Collapse of the traditional pyramid of education

In Britain, the natural cycle of reproduction of mathematics as a cultural system and a professional community is broken.

> . . . the performance of more able pupils had collapsed; the numbers taking A-level maths were falling dramatically; those with top grades were "increasingly innumerate and even ineducable"; the shortage of qualified maths teachers had reached "dangerous" levels; national test results were grossly inflated; and postgraduates with a PhD in maths from a British university were now "largely unemployable" in British universities. (The Daily Telegraph, 28 June 2005).

Rebranding mathematics

The key to the success of our enterprize
is the aggressive marketing
of the religious product.
Rabbi of the Reformist Synagogue,
Irvine, California, c. 1990

Why not rebrand mathematics as a tool of personal development and a spiritually enhancing activity?

Why not try to create an up-market brand of maths learning, for the top 5% who still need it?

Rebranding mathematics

The new approach to mathematical education will not work unless we know:

- What are mathematical abilities?
- What is the nature of mathematical intuition?
- What children actually do when they learn mathematics?
- What mathematicians actually do when they do mathematics?